### 6.4.1 EXERCISES

For a link to all of the additional resources available for this section, click OSttS Chapter 6 materials.

In Exercises 1 - 24, solve the equation analytically.

For help with these exercises, click on the resource below:

- Solving equations involving logarithmic functions

1. $\log(3x-1) = \log(4-x)$
2. $\log_2\left(x^3\right) = \log_2(x)$
3. $\ln\left(8-x^2\right) = \ln(2-x)$
4. $\log_5\left(18-x^2\right) = \log_5(6-x)$
5. $\log_3(7-2x) = 2$
6. $\log_{\frac{1}{2}}(2x-1) = -3$
7. $\ln\left(x^2-99\right) = 0$
8. $\log(x^2-3x) = 1$
9. $\log_{125}\left(\dfrac{3x-2}{2x+3}\right) = \dfrac{1}{3}$
10. $\log\left(\dfrac{x}{10^{-3}}\right) = 4.7$
11. $-\log(x) = 5.4$
12. $10\log\left(\dfrac{x}{10^{-12}}\right) = 150$
13. $6 - 3\log_5(2x) = 0$
14. $3\ln(x) - 2 = 1 - \ln(x)$
15. $\log_3(x-4) + \log_3(x+4) = 2$
16. $\log_5(2x+1) + \log_5(x+2) = 1$
17. $\log_{169}(3x+7) - \log_{169}(5x-9) = \dfrac{1}{2}$
18. $\ln(x+1) - \ln(x) = 3$
19. $2\log_7(x) = \log_7(2) + \log_7(x+12)$
20. $\log(x) - \log(2) = \log(x+8) - \log(x+2)$
21. $\log_3(x) = \log_{\frac{1}{3}}(x) + 8$
22. $\ln(\ln(x)) = 3$
23. $(\log(x))^2 = 2\log(x) + 15$
24. $\ln(x^2) = (\ln(x))^2$

In Exercises 25 - 30, solve the inequality analytically.

25. $\dfrac{1-\ln(x)}{x^2} < 0$
26. $x\ln(x) - x > 0$
27. $10\log\left(\dfrac{x}{10^{-12}}\right) \geq 90$
28. $5.6 \leq \log\left(\dfrac{x}{10^{-3}}\right) \leq 7.1$
29. $2.3 < -\log(x) < 5.4$
30. $\ln(x^2) \leq (\ln(x))^2$

In Exercises 31 - 34, use your calculator to help you solve the equation or inequality.

31. $\ln(x) = e^{-x}$

32. $\ln(x) = \sqrt[4]{x}$

33. $\ln(x^2 + 1) \geq 5$

34. $\ln(-2x^3 - x^2 + 13x - 6) < 0$

35. Since $f(x) = e^x$ is a strictly increasing function, if $a < b$ then $e^a < e^b$. Use this fact to solve the inequality $\ln(2x + 1) < 3$ without a sign diagram. Use this technique to solve the inequalities in Exercises 27 - 29. (Compare this to Exercise 46 in Section 6.3.)

36. Solve $\ln(3 - y) - \ln(y) = 2x + \ln(5)$ for $y$.

37. In Example 6.4.4 we found the inverse of $f(x) = \dfrac{\log(x)}{1 - \log(x)}$ to be $f^{-1}(x) = 10^{\frac{x}{x+1}}$.

    (a) Show that $\left(f^{-1} \circ f\right)(x) = x$ for all $x$ in the domain of $f$ and that $\left(f \circ f^{-1}\right)(x) = x$ for all $x$ in the domain of $f^{-1}$.

    (b) Find the range of $f$ by finding the domain of $f^{-1}$.

    (c) Let $g(x) = \dfrac{x}{1 - x}$ and $h(x) = \log(x)$. Show that $f = g \circ h$ and $(g \circ h)^{-1} = h^{-1} \circ g^{-1}$. (We know this is true in general by Exercise 31 in Section 5.2, but it's nice to see a specific example of the property.)

38. Let $f(x) = \dfrac{1}{2} \ln\left(\dfrac{1 + x}{1 - x}\right)$. Compute $f^{-1}(x)$ and find its domain and range.

39. Explain the equation in Exercise 10 and the inequality in Exercise 28 above in terms of the Richter scale for earthquake magnitude. (See Exercise 75 in Section 6.1.)

40. Explain the equation in Exercise 12 and the inequality in Exercise 27 above in terms of sound intensity level as measured in decibels. (See Exercise 76 in Section 6.1.)

41. Explain the equation in Exercise 11 and the inequality in Exercise 29 above in terms of the pH of a solution. (See Exercise 77 in Section 6.1.)

42. With the help of your classmates, solve the inequality $\sqrt[n]{x} > \ln(x)$ for a variety of natural numbers $n$. What might you conjecture about the "speed" at which $f(x) = \ln(x)$ grows versus any principal $n^{\text{th}}$ root function?

**Checkpoint Quiz 6.4**

1. Solve for $x$: $\log_2(3-x)+\log_2(1-x)=3$

2. Solve for $x$: $(\log(x-2))^2 \geq \log(x-2)+12$

For worked out solutions to this quiz, click the links below:

- Quiz Solution Part 1
- Quiz Solution Part 2

### 6.4.2 Answers

1. $x = \frac{5}{4}$
2. $x = 1$
3. $x = -2$
4. $x = -3,\ 4$
5. $x = -1$
6. $x = \frac{9}{2}$
7. $x = \pm 10$
8. $x = -2,\ 5$
9. $x = -\frac{17}{7}$
10. $x = 10^{1.7}$
11. $x = 10^{-5.4}$
12. $x = 10^{3}$
13. $x = \frac{25}{2}$
14. $x = e^{3/4}$
15. $x = 5$
16. $x = \frac{1}{2}$
17. $x = 2$
18. $x = \frac{1}{e^3 - 1}$
19. $x = 6$
20. $x = 4$
21. $x = 81$
22. $x = e^{e^3}$
23. $x = 10^{-3},\ 10^{5}$
24. $x = 1,\ x = e^2$
25. $(e, \infty)$
26. $(e, \infty)$
27. $\left[10^{-3}, \infty\right)$
28. $\left[10^{2.6}, 10^{4.1}\right]$
29. $\left(10^{-5.4}, 10^{-2.3}\right)$
30. $(0, 1] \cup [e^2, \infty)$
31. $x \approx 1.3098$
32. $x \approx 4.177,\ x \approx 5503.665$
33. $\approx (-\infty, -12.1414) \cup (12.1414, \infty)$
34. $\approx (-3.0281, -3) \cup (0.5, 0.5991) \cup (1.9299, 2)$
35. $-\dfrac{1}{2} < x < \dfrac{e^3 - 1}{2}$
36. $y = \dfrac{3}{5e^{2x} + 1}$
38. $f^{-1}(x) = \dfrac{e^{2x} - 1}{e^{2x} + 1} = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$. (To see why we rewrite this in this form, see Exercise 51 in Section 11.10.) The domain of $f^{-1}$ is $(-\infty, \infty)$ and its range is the same as the domain of $f$, namely $(-1, 1)$.